22 $1.27 \times 10^{-5}$ 40 D 42 B

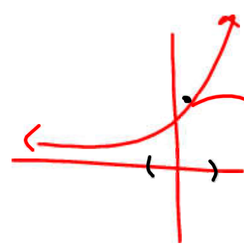
41, 42, 33,

33 $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$

$f(x) = e^x$

$P_3(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + R_3 \leftarrow$ on $-0.1 < x < 0.1$

$|R_3| \leq \frac{\boxed{f^{(4)}(c)}}{4!} x^4 \rightarrow f^{(4)}(x) = e^x$



$\leq \frac{e^{0.1}}{4!}(0.1)^4$ $\rightarrow e^{0.1} = M$

$\leq 4.605 \times 10^{-6}$

( **41. Multiple Choice** Let $\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!} = x + x^2 + \frac{x^3}{2!} + \cdots$ be the $+ \frac{x^{12}}{11!}$ Maclaurin series for $f(x)$. Which of the following is $f^{(12)}(0)$, the 12th derivative of $f$ at $x = 0$? E

(A) 1/11! (B) 1/12! (C) 0 (D) 1 (E) 12

$$\boxed{\frac{f^{(12)}(0)}{12!}} x^{12} = \frac{x^{12}}{11!}$$

$$\frac{\cancel{12}}{\cancel{12} \cdot 11!} x^{12} = \frac{x^{12}}{11!}$$

**42. Multiple Choice** Let $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

be the Maclaurin series for $\cos x$. Which of the following gives the smallest value of $n$ for which $|P_n(x) - \cos x| < 0.01$ for all $x$ in the interval $[-\pi, \pi]$? B error < 0.01

(A) 12 (B) 10 (C) 8 (D) 6 (E) 4

$$|R_n(x)| \le \frac{f^{(n+1)}(c)}{n+1!} x^{n+1} \quad \text{on } [-\pi, \pi]$$

M=1

$$\le \frac{1}{(n+1)!} \cdot \pi^{n+1} < 0.01$$

$$y1 = \frac{\pi^{n+1}}{(n+1)!}$$

Table:

| x | y |
|---|---|
| 9 | 0.02581 |
| 10 | 0.00737 |

B

use $P_{10}$

**34.** ***A Cubic Approximation*** Use the Taylor polynomial of order 3 to find the *cubic approximation* of $f(x) = 1/(1 - x)$ at $x = 0$. Give an upper bound for the magnitude of the approximation error for $|x| \leq 0.1$.

$$\frac{1}{1-x} \approx 1 + x + x^2 + x^3$$

# More 9.4 - Convergence

## Tests for Convergence or Divergence

- nth Term Test for Divergence

  nth term must $\to 0$ as $n \to \infty$ or series diverges.

  ☆ Is $\lim_{n\to\infty} a_n = 0$? $\xrightarrow{\text{no}}$ diverges

  $\downarrow$ yes or maybe

  inconclusive - do more tests

Ex) $\sum_{n=1}^{\infty} \frac{2^n}{n+1}$ converge or diverge?

$$\lim_{n\to\infty} \frac{2^n}{n+1} \text{ "}\frac{\infty}{\infty}\text{"} = \lim_{n\to\infty} \frac{2^n \cdot \ln 2}{1} = \infty \quad \therefore \text{ series diverges}$$

## Direct Comparison Test

Compare to a known series: $e^x$, $\cos x$, $\sin x$, P-series

Let $\sum a_n$ be a series with no negative terms

Then: ☆ $\sum a_n$ converges ~~if there~~ is another series $\sum b_n$ that also converges and $a_n \le b_n$ for all $n < N$ for some integer $N$.

☆ $\sum a_n$ diverges if there is another series $\sum b_n$ that also diverges and $a_n \ge b_n$ for all $n < N$ for some integer $N$.

Ex) $$\sum_{n=0}^{\infty} \frac{x^n}{(n!)^2} = 1 + \frac{x^1}{(1!)^2} + \frac{x^2}{(2!)^2} + \frac{x^3}{(3!)^2} + \cdots$$

Compare to $e^x$. $1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

our series $< e^x$ series and $e^x$ converges,

$\therefore$ our series converges as well.

② $\sum_{n=0}^{\infty} \frac{x^{2n}}{(n!)^2}$ Prove that it converges.

$$= 1 + \frac{x^2}{(1!)^2} + \frac{x^4}{(2!)^2} + \frac{x^6}{(3!)^2} + \cdots$$

Compare to $e^{x^2} = 1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots$

$e^x$ converges, so $e^{x^2}$ converges as well.

our series $< e^{x^2}$ $\therefore$ it must converge as well.

HW: Ch 8 Graded HW, p300 #34